Reliability analysis in vehicle collision with bridge pier

Keivan Shariatmadar, Gert de Cooman, Pieter Baekeland, Erik Quaeghebeur and Etienne Kerre

Abstract

Much work has been done in bridge design specification via a set of structural design standards called *Eurocodes* to cover the design of all types of structures. We analyse the accidental force on a bridge pier when it is hit by vehicles in order to assess the reliability of a bridge. The force that comes from a vehicle—called *vehicle impact force*—is not deterministic and it depends on some *uncertain* parameters, such as the mass of the vehicle and its speed on impact. All the data and uncertainty models for the parameters are given by Eurocode 1. In this paper we analyse the force that is affected by these parameters. For doing that we consider two kinds of problems where in the both problems this force is a function on a *distance*—the distance between the bridge pier and the side of a road passing under the bridge. One of the problems proposes a design force as a function of the distance—called *reliable distance*—using a strength condition, the condition on the design forces and the other one suggests a tool for obtaining an economical optimum distance—called *cost-optimal distance*—by taking into account the optimum economical costs—the cost of bulding and repairing the bridge and human life. In both problems, we consider the safety of the distance where affects dynamic and static design forces and the impact force of vehicle which is not a constant. We show how reliable are Eurocodes by comparing these two distances calculated in two different problems. In other words, through these two problems/criteria we show the danger of using the data represented via Eurocodes for the parameters. In addition, we found linear functions on the distance and the (dynamic and static) design forces of the bridge.

Keywords

Vehicle impact force, Bridge design force, Decision theory, Eurocode, Bridge pier

1. Introduction

In bridge design, it is very important to find the distance between the pier and the side of the road under the bridge such that ensures structural integrity, or equivalently, which satisfies the condition on the design force (ILES, 2010; Baekeland, 2011). One of the risks is collison to the pier. To prevent this risk one can avoid the collision by finding a reliable distance between the side of a road and bridge pier such that there is no collision with 99% probability (Jana and Karel, 2008; Kulicki, 2006; Baekeland, 2010a; Baekeland, 2010b). On the other hand, the effect of vehicle impacts on the integrity of a bridge is a complex issue, involving dynamic effects, non-linear material behaviour, stiffness of the vehicle and the bridge, the presence of road restraining systems, shape and roughness of the terrain, mass and speed of the vehicle, impact angle, deceleration, which are not *certainly known* and it is assumed that they are independent.

The paper is organized as follows. We work with three-span bridge which is explained in the Section 2. By analysing and solving these two problems we show that the data (Eurocode) are not reliable. In this paper we consider the bridge collision problem where the distance can be obtained from two kinds of criteria/problems. The two design criteria and methodology are given in Section 3. Furthermore, we find linear relations between this distance and bridge design forces and the results are briefly explained in the Section 4. We present a summary in Section 5.

2. Problem statement: Bridge collision problem

We consider a situation in which a vehicle collides with the pillar of a bridge over the road that vehicle was travelling on. See Figure 1 for a schematic drawing of the accident situation and the type of bridge (three-span bridge).

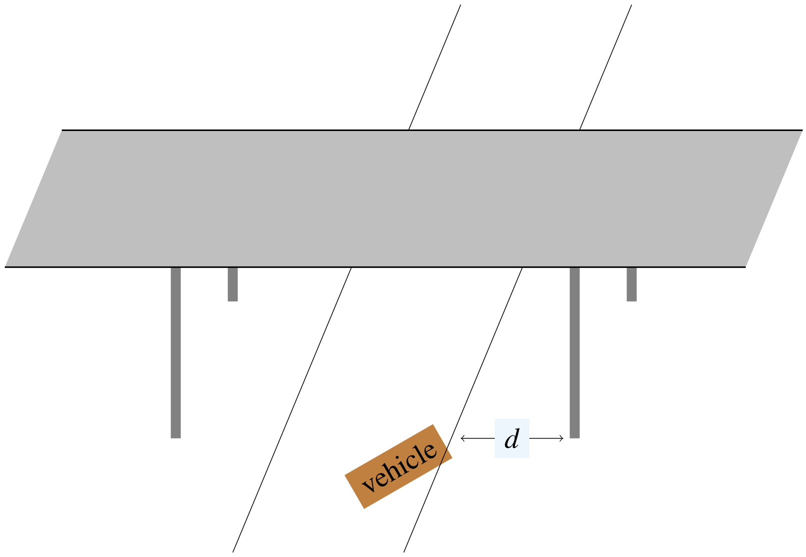

Figure 1 - Bridge collision problem

In Figure 2 the parameters and the relations are shown schematically in details where: The mass of the vehicle is m. The speed at which it leaves the carriageway is v_0. Its average deceleration while off the carriageway is a. The angle is α. The distance travelled in the deceleration period is r. The distance of the obstacle perpendicular to the carriageway is d. The stiffness of the vehicle is k. F_{d} is the dynamic design force with projections F_{dx} and F_{dy}.

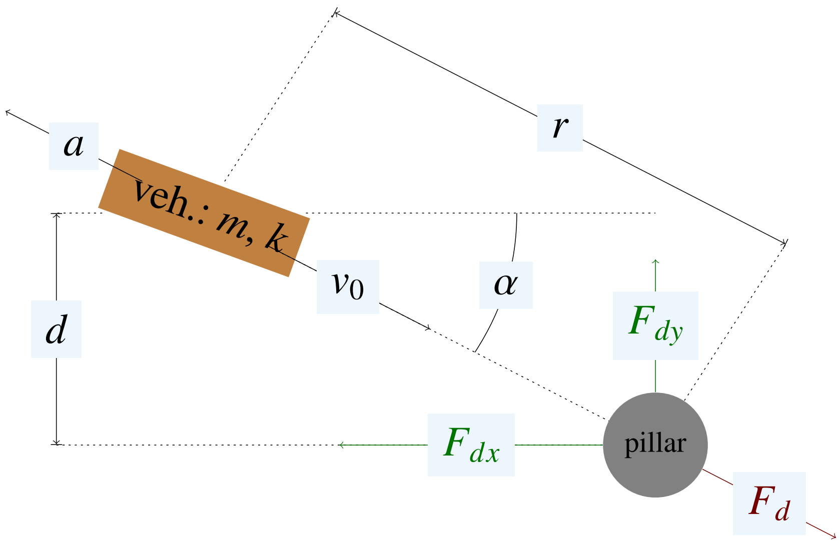

Figure 2 – Top view of the pillar

In this problem, some of the parameters $Y \coloneqq (m, v_0, a, \alpha)$ are assumed to be uncertain. This uncertainty is represented by probability distributions (de Finetti, 1974). The parameters are assumed to be independent.

In recent work (Shariatmadar et al., 2010) we found the cost-optimal distance d between the road and the bridge pillar that ensures structural integrity, i.e., we found the distance that minimises a given cost function while still satisfying in the constraints about the design forces where there is uncertainty about the parameters which is called (non-linear) constrained optimization problem under uncertainty. A more general and theoretical of this problem is given by (Quaeghebeur, Shariatmadar and de Cooman, 2010). In the paper (Shariatmadar et al., 2010), we realise that some of the results were likely unrealistic owing to the data given by TECHNUM-TRACTEBEL company and based on Eurocode (EN 1991-1-7 : Eurocode 1, 2006; EN 1991-2: Eurocode 1, 2003). This lead us to start to investigate about the reliability of the data by defining the two criteria which is explained in detail in this paper. The goal is to make a decision about the distance by using two design criteria to see how reliable are the design and the data given by Eurocode and compare the results.

3. Methodology and decision criteria

The two decision criteria that we discussed above are defined as, first: Minimising expected utility (Troffaes, 2008); what is the best decision which minimises the expected loss (cost), i.e., decision z_1 is better than decision z_2 if and only if expected value of a given utility function coresponding to z_1—$E(z_1)$—is strictly greater than the expected value of given utility function coresponding to z_2—$E(z_2)$, we write: $z_1 \succ z_2 \Leftrightarrow E(z_1) > E(z_2)$.

After applying this criterion to the bridge collision problem we obtain the following decision problem: what is corresponding distance $d \coloneqq d_{\text{opt}}$ between the pillar and the side of the road to a given dynamic impact force (design force) which minimises a cost function?

Second: Strength condition on integrity: what is reliable distance fulfilling some condition/conditions, i.e., what is the corresponding distance $d \coloneqq d_{\mathrm{const}}$ such that the *impact force* of the vehicle is smaller than the bridge *design force* in 99% of the impact cases? We call the first problem: Maximinity method and the second problem: Strength method. These two problems are explained in sections 3.1 and 3.2.

3.1 The first problem: Maximinity method

We define a cost function which is function of vehicle impact force F_{veh}. The cost is about the cost of building the bridge including economical loss, human life and repairing cost after the damage. This cost function is minimised for a given dynamic impact force. The dynamic impact force expressed in an inequality which is one of the important condition on the bridge design: the vehicle impact force F_{veh} has to be smaller than the bridge dynamic design force F_{d},

$$F_{\mathrm{veh}} < F_{\mathrm{d}} \quad \text{(point-wise)} \Leftrightarrow (F_{\mathrm{veh}}\, cos(\alpha) < F_{\mathrm{dx}} \quad \text{and} \quad F_{\mathrm{veh}}\, sin(\alpha) < F_{\mathrm{dy}})$$

Equation 1

Where $F_{\mathrm{veh}} \coloneqq \sqrt{mk\, v_0 - 2ad/sin(\alpha)}$ and $F_{\mathrm{d}} \coloneqq \varphi_{\mathrm{d}} F_{\mathrm{de}}$ (point-wise), F_{de} is the equivalent static design force acting on the obstacle and $\varphi_{\mathrm{d}} \in [0.6,2.4]$ is the dynamic amplification factor. Then a random variable (vector) is define as $Y \coloneqq (m, v_0, a, \alpha) \in \mathbb{R}^4$ which is a quadruple of uncertain variables (mass of the vehicle, the speed, average deceleration and the angle α, respectively).

In what follows, we consider two categories for the vehicles, based on the type of vehicle and the speed when leaving the carriageway. Some of the parameters have a probability distribution according to Table 1 of the Eurocode (EN 1991-1-7 : Eurocode 1, 2006). We use the normal distribution $\mathcal{N}(\mu, \sigma)$, the lognormal distribution $\mathcal{L}(\mu, \sigma)$, and the Rayleigh distribution $\mathcal{R}(\mu, \sigma)$, each of which is characterised by a mean μ and standard deviation σ and must be truncated to within the given reasonable interval.

The equivalent static design forces due to the impact of vehicles on members supporting structures over or adjacent to roadways is given in Table 1. These design values are independent of the distance d.

Parameter	Highway lorry	Urban Lorry
v_0 [km/h]	$[50{,}100]$ $\mathcal{L}(80{,}10)$	$[30{,}70]$ $\mathcal{L}(40{,}8)$
m [t]	$[12{,}40]$ $\mathcal{N}(20{,}12)$	$[12{,}40]$ $\mathcal{N}(20{,}12)$
α [°]	$[8{,}45]$ $\mathcal{R}(19{,}10)$	$[8{,}45]$ $\mathcal{R}(10{,}10)$
a [m/s^2]	$[1{,}5]$ $\mathcal{L}(4{,}1.3)$	$[1{,}5]$ $\mathcal{L}(4{,}1.3)$
k [kN/m]	300	300

Table 1 – Parameter value information for the different vehicle–speed categories

In Table 2 the design forces of the bridge are given,

Category of traffic	F_{dex} [kN]	F_{dey} [kN]
Motorways, country national and main roads	1000	500
Roads in urban area	500	250

Table 2 – Indicative equivalent static design forces.

Now, the first problem is summerized as a decision problem: decide which d to choose in the face of our uncertainty about Y . When the uncertainty about Y is modelled probabilistically, the approach usually followed is to choose (the action) d which minimises the expected cost $E[C_{\mathrm{d}}(F_{\mathrm{veh}})]$, where E is the expectation operator, C_{d} is cost utility in function of vehicle impact force—F_{veh} .

We solve this by performing a Monte Carlo simulation (Harrison, 2010), which is used to compute for generating probability distributions of variables that depend on other variables or parameters represented as probability distributions. In its most straightforward form, a Monte Carlo simulation usually assumes that input parameters are independent. In our problem, by discretising the distance d in a realistic range for each given φ_{d}, we calculate the corresponding expected cost function $E[C_{\mathrm{d}}(F_{\mathrm{veh}})]$. To do this, because of the independency, we perform a simple Monte Carlo analysis to obtain the cost function $C_{\mathrm{d}}(F_{\mathrm{veh}})$. Then through calculating the mean, we find $E[C_{\mathrm{d}}(F_{\mathrm{veh}})]$. Finally, we obtain the economical optimum distance d_{opt} by minimising the expectation of the cost function over d, i.e., $\mathrm{argmin}_{\mathrm{d}}\, E[C_{\mathrm{d}}(F_{\mathrm{veh}})]$. The cost function C_{d} is defined in the next section.

3.1.1 Cost function

In this paper, the cost of the building of a bridge, denoted by $K_{\mathrm{b}}(d)$, which is proportional to the square of the span of the bridge, i.e.,

$$K_{\mathrm{b}}(d) \coloneqq c(35 + d)^2$$

Equation 2

where c is a constant (here $c = 1$). A typical span is 35 [m] plus the distance d of the pillar to the carriageway. When the impact force of a colliding vehicle on a bridge pillar is smaller than the design value, there will be some repair costs which is denoted by $K_{\mathrm{r}}(d)$. The repair costs depend on a factor s which is the ratio of the impact force to the design force and that is the precise ratio. The factor s for the forces parallel and perpendicular to the carriageway is shown respectively by s_{x} and s_{y}, and is defined as

$$\boldsymbol{s_y \coloneqq F_{veh} sin(\alpha)/F_{dy}} \quad \text{and} \quad \boldsymbol{s_x \coloneqq F_{veh}\, cos(\alpha)/F_{dx}}$$

Equation 3

We assume that the repair cost is proportional to the factor s and the repair cost is 5% of the building cost—$K_{\mathrm{b}}(d)$—for $s = 1$ then,

$$K_{\mathrm{r}}(d) \coloneqq sK_{\mathrm{b}}(d)/20 \Leftrightarrow K_{\mathrm{r}}^{\mathrm{x}}(d) \coloneqq s_{\mathrm{x}} K_{\mathrm{b}}(d)/20\,,\ K_{\mathrm{r}}^{\mathrm{y}}(d) \coloneqq s_{\mathrm{y}} K_{\mathrm{b}}(d)/20$$

Equation 4

The cost of damage rises very sharply when the hitting force is bigger than the design force. For s bigger than 1.5 one will not only end up with rebuilding the whole bridge but also with economical loss and human life loss. When the impact force of a colliding vehicle on a bridge pillar is larger than 1.15 to 1.20 times the design value, the entire bridge is assumed to be lost and the economic damage with $K_{\mathrm{e}}(d)$ is:

$$K_{\mathrm{e}}(d) \coloneqq K_{\mathrm{b}}(d)s^{20}/20 \Leftrightarrow K_{\mathrm{e}}^{\mathrm{x}}(d) \coloneqq K_{\mathrm{b}}(d)s_{\mathrm{x}}^{20}/20\,,\ K_{\mathrm{e}}^{\mathrm{y}}(d) \coloneqq K_{\mathrm{b}}(d)s_{\mathrm{y}}^{20}/20$$

Equation 5

The power 20 indicates only a sharp rise of the total cost. Finally, the cost function

$$C_{\mathrm{d}}^{\mathrm{x}}(F_{\mathrm{veh}}) \coloneqq K_{\mathrm{b}}(d) + \begin{cases} K_{\mathrm{r}}^{\mathrm{x}}(d), & s_{\mathrm{x}} \le 1 \\ K_{\mathrm{e}}^{\mathrm{x}}(d), & s_{\mathrm{x}} > 1 \end{cases}$$

$$C_{\mathrm{d}}^{\mathrm{y}}(F_{\mathrm{veh}}) \coloneqq K_{\mathrm{b}}(d) + \begin{cases} K_{\mathrm{r}}^{\mathrm{y}}(d), & s_{\mathrm{y}} \le 1 \\ K_{\mathrm{e}}^{\mathrm{y}}(d), & s_{\mathrm{y}} > 1 \end{cases}$$

Equation 6

3.2 The second problem: Strength method

The strength approach deals with finding a design force as a function of the distance d, such that in case of collision, the impact force is smaller than the design force in 99% of the cases. For a given dynamic amplification factor φ_{d}, we find the distance such that the conditions in (1) are fulfilled. This is done just by solving the following equation to obtain d_{const} with a numerical algotithem,

$$P(F_{\mathrm{veh}} < F_{\mathrm{d}}) - 0.99 = 0$$

Equation 7

where P is notation for the probability.

4. Vehicle impact on bridge pillar: Results

In the first problem, the corresponding distance d_{opt} depends mainly on a cost condition. In the second problem, the distance d_{const} depends only on a strength condition. In both problems the uncertain variables are given as probability distributions by (EN 1991-1-7 : Eurocode 1, 2006).

We propose a design force as a function of the distance: d_{const} —using a strength condition and d_{opt} —suggests an optimisation tool for an economical optimum distance.

4.1 Highway lorry

In the absence of a dynamic analysis, the dynamic amplification factor for the elastic response may be assumed to be equal to 1.40 (EN 1991-1-7 : Eurocode 1, 2006). If the structure is assumed to respond elastically and the load is realised as a step function, the dynamic amplification factor for the elastic response has a upper bound of 2.00 (EN 1991-1-7 : Eurocode 1, 2006). We plot the dynamic impact force (parallel and perpendicular) as a function of the distance d obtained by two methods: the strength method (d_{const}) and the maximinity method (d_{opt}).

4.1.1 Design forces parallel to the carriageway (x-direction)

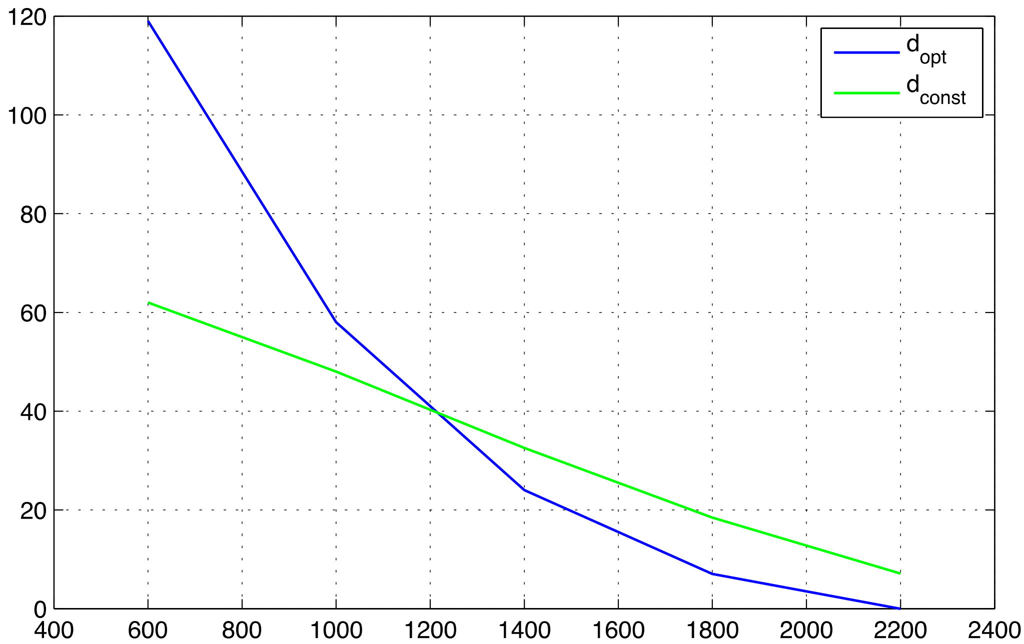

Figure 3 - Dynamic impact force parallel to carriageway

The results that are obtained with two methods are:

4.1.1.1 Strength method (parallel force)

Using the strength method we find a linear relationship between the distance d and the dynamic impact force F_{dx},

$$F_{\mathrm{dx}} = 2358 - 28 d_{\mathrm{const}}$$

Equation 8

At the lower limit of the dynamic amplification factor the equivalent static design force can be written as (force in [kN] and distance in [m]),

$$F_{\mathrm{dex}} = 1684 - 20 d_{\mathrm{const}} = F_{\mathrm{dx}}/1.40$$

Equation 9

For a design force of 1000 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 34 [m]!

At the upper limit of the dynamic amplification factor the equivalent static design force can be written as (force in [kN] and distance in [m]),

$$F_{\mathrm{dex}} = 1179 - 14d_{\mathrm{const}} = F_{\mathrm{dx}}/2.00$$

Equation 10

For a design force of 1000 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 13m.

The distance d from pillar to road can also be chosen in accordance with what the usage of the area (between road and pillar). The *functionality distance* d_{fun} is the sum of the widths of the emergency lane, the dewatering facilities and the maintenance road or inspection path. The functionality distance varies from 5 to 10m.

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived by using $\mathrm{F}_{\mathrm{dx}} = 2358 - 28\mathrm{d}_{\mathrm{const}}$ (10):

$$F_{\mathrm{dx}} = 2078 \text{ kN}$$

and then by $\mathrm{F}_{\mathrm{dex}} = 1684 - 20\mathrm{d}_{\mathrm{const}} = \mathrm{F}_{\mathrm{dx}}/1.40$ (11**)** and $\mathrm{F}_{\mathrm{dex}} = 1179 - 14\mathrm{d}_{\mathrm{const}} = \mathrm{F}_{\mathrm{dx}}/2.00$ (12),

$$F_{\mathrm{dex}} = 1484 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dex}} = 1039 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) is in most cases **not safe to use**. Only when the pillar is very stiff (the load is a step function) the design force of 1000 [kN] is adequate for distances greater than 10 [m].

4.1.1.2 Maximinity method (parallel force)

For a design force of 1000 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 1.40$ the distance d must be greater than 24 [m] !

For a design force of 1000 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 2.00$ the distance d must be greater than 3.5 [m].

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived using the cost optimisation method: $F_{\mathrm{dx}} = 1730$ [kN] and then

$$F_{\mathrm{dex}} = 1236 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dex}} = 865 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

For a distance d smaller than 40m the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning. At the distance $d = d_{\mathrm{fun}}$ the reduction in equivalent static design force is about 20%.

4.1.2 Design forces perpendicular to the carriageway (y-direction)

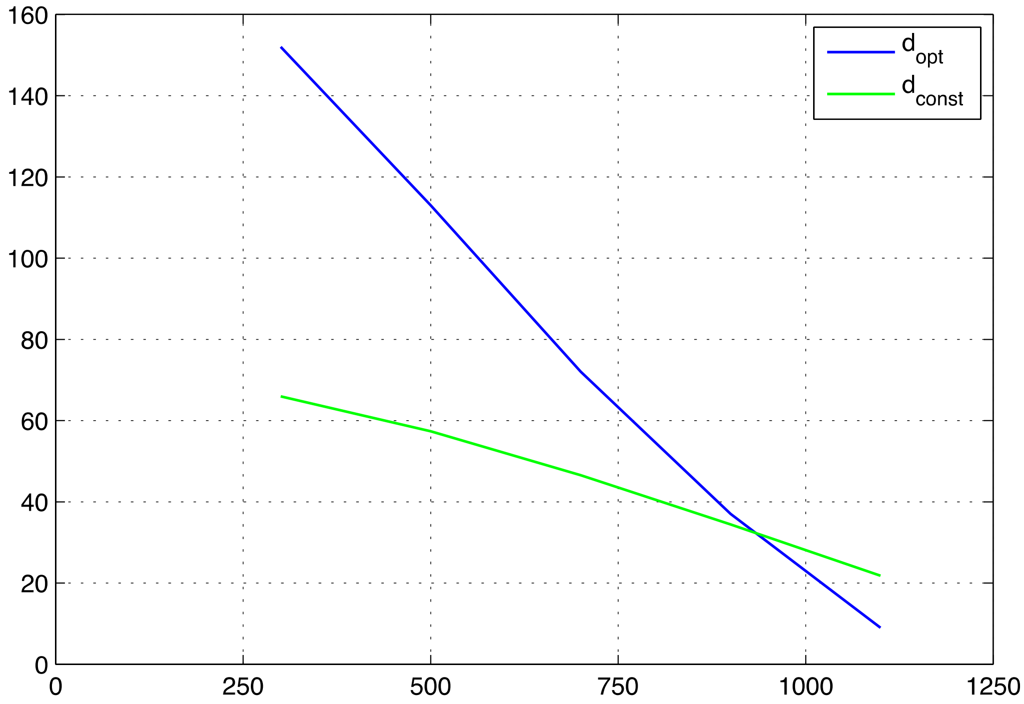

Figure 4 - Dynamic impact force perpendicular to the carriageway

Figure 4 represents the dynamic impact force in function of the distance d obtained by the two methods: strength method (d_{const}) and the maximinity method (d_{opt})

4.1.2.1 Strength method (perpendicular force)

By using the strength method we find a linear relationship between the distance d and the dynamic impact force F_{dy}

$$\boldsymbol{F}_{\text{dy}} = \mathbf{1508 - 18}\boldsymbol{d}_{\text{const}}$$

Equation 11

At the lower limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$\boldsymbol{F}_{\text{dey}} = \mathbf{1077 - 13}\boldsymbol{d}_{\text{const}}$$

Equation 12

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 44 [m]!

At the upper limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$F_{\text{dey}} = 754 - 9d_{\text{const}}$$

Equation 13

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 28 [m]!

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived by using $\mathrm{F}_{\text{dy}} = 1508 - 18\mathrm{d}_{\text{const}}$ (13), $\mathrm{F}_{\text{dey}} = 1077 - 13\mathrm{d}_{\text{const}}$ (14) and $\mathrm{F}_{\text{dey}} = 754 - 9\mathrm{d}_{\text{const}}$ (15):

$$F_{\text{dy}} = 1328 \text{ [kN]}$$

$$F_{\text{dey}} = 947 \text{ [kN]} \quad : \quad \varphi_{\text{d}} = 1.40$$

$$F_{\text{dey}} = 664 \text{ [kN]} \quad : \quad \varphi_{\text{d}} = 2.00$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) **is not safe to use**.

4.1.3 Maximinity method (perpendicular force)

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 1.40$ the distance d must be greater than 77 [m]!

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 2.00$ the distance d must be greater than 22 [m]!

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived using the cost optimisation method: $F_{\mathrm{dy}} = 1093$ [kN] then

$$F_{\mathrm{dey}} = 781 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dey}} = 547 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

For a distance d smaller than 41 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning.

At the distance $d = d_{\mathrm{fun}}$ the reduction in equivalent static design force is about 20%.

4.2 Urban lorry

4.2.1 Design forces parallel to the carriageway (x-direction)

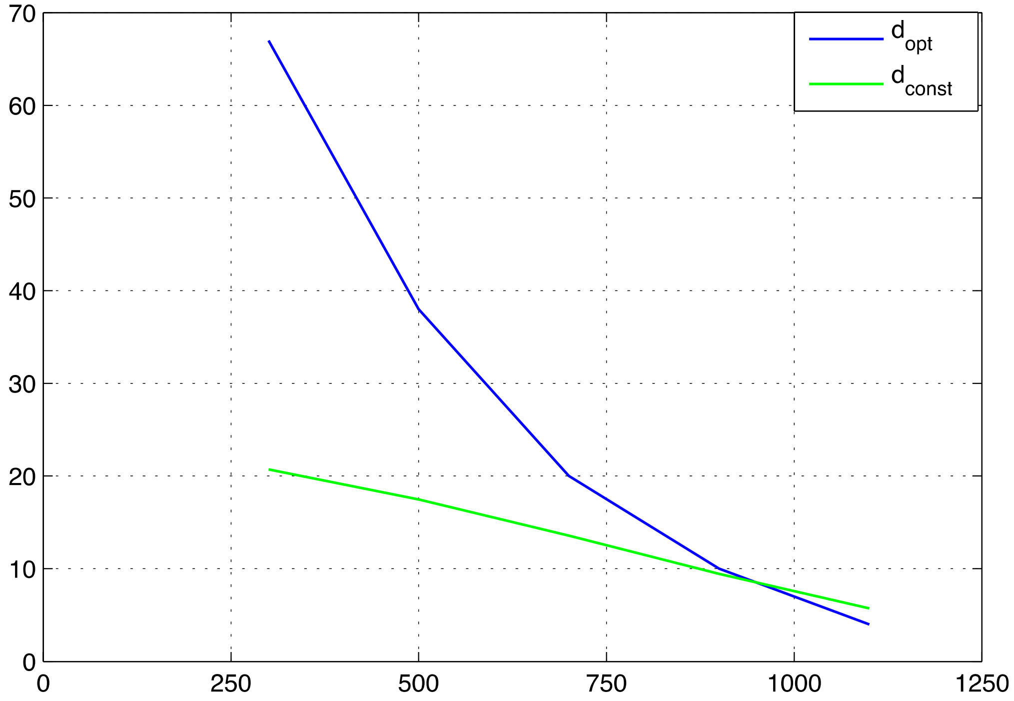

Figure 5 - Dynamic impact force parallel to the carriageway

Figure 5 represents the dynamic impact force as a function of the distance d obtained by the strength method (d_{const}) and the maximinity method (d_{opt}),

4.2.1.1 Strength method (parallel force)

By using the strength method we find a linear relationship between the distance d and the dynamic impact force F_{dx}:

$$\boldsymbol{F}_{\mathrm{dx}} = \mathbf{1402} - \mathbf{52}\boldsymbol{d}_{\mathrm{const}}$$

Equation 14

At the lower limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$F_{\mathrm{dex}} = 1001 - 37 d_{\mathrm{const}} = F_{\mathrm{dx}}/1.40$$

Equation 15

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 13 [m].

At the upper limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$\boldsymbol{F}_{\mathrm{dex}} = \mathbf{701} - \mathbf{26}\boldsymbol{d}_{\mathrm{const}} = \boldsymbol{F}_{\mathrm{dx}}/\mathbf{2.00}$$

Equation 16

For a design force of of 500 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 8 [m].

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived, $F_{\mathrm{dx}} = 882$ [kN] then

$$F_{\mathrm{dex}} = 631 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dex}} = 441 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) is in most cases safe to use. The design force of 500 [kN] is ok for distances greater than 8 to 13 [m] (depending on the dynamic response of the pillar)

4.2.1.2 Maximinity method (parallel force)

For a design force of 500 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 1.40$ the distance d must be greater than 22 [m]!

For a design force of of 500 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 2.00$ the distance d must be greater than 7.5 [m].

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived using the cost optimisation method: $F_{\mathrm{dx}} = 929$ [kN] then

$$F_{\mathrm{dex}} = 664 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dex}} = 465\ [\mathrm{kN}] \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

For a distance d smaller than 8 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning. At the distance $d = 5$[m] the reduction in equivalent static design force is about 10%.

4.2.2 Design forces perpendicular to the carriageway (y-direction)

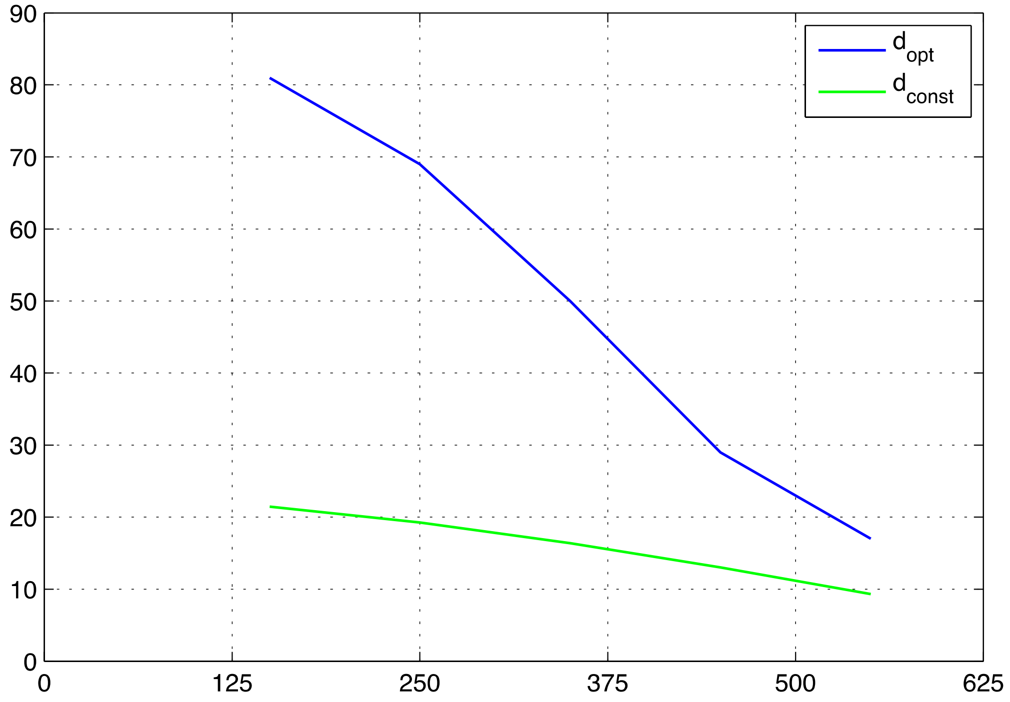

Figure 6 - Dynamic impact force perpendicular to the carriageway

Figure 6 represents the dynamic impact force in function of the distance d obtained by the strength method (d_{const}) and the maximinity method (d_{opt}),

4.2.2.1 Strength method (perpendicular force)

By using the strength method we find a linear relationship between the distance d and the dynamic impact force F_{dy}:

$$\boldsymbol{F}_{\mathbf{dy}} = \mathbf{867} - \mathbf{33}\boldsymbol{d}_{\mathbf{const}}$$

Equation 17

At the lower limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$\boldsymbol{F}_{\mathrm{dey}} = \mathbf{619} - \mathbf{23}\boldsymbol{d}_{\mathrm{const}} = \boldsymbol{F}_{\mathrm{dy}}/\mathbf{1.40}$$

Equation 18

For a design force of 250 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 16 [m]!

At the upper limit of the dynamic amplification factor the equivalent static design force is given by (force in [kN] and distance in [m]),

$$F_{\mathrm{dey}} = 433 - 16d_{\mathrm{const}} = F_{\mathrm{dy}}/2.00$$

Equation 19

For a design force of 250 [kN] (EN 1991-2: Eurocode 1, 2003) the distance d must be greater than 11 [m].

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived,

$$F_{\mathrm{dy}} = 537 \text{ [kN]}$$

$$F_{\mathrm{dey}} = 389 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dey}} = 273 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) **is not safe to use**.

4.2.2.2 Maximinity method (perpendicular force)

Using the cost optimisation method the following results are obtained,

For a design force of 250 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 1.40$ the distance d must be greater than 52 [m]!

For a design force of 250 [kN] (EN 1991-2: Eurocode 1, 2003) and $\varphi_{\mathrm{d}} = 2.00$ the distance d must be greater than 26 [m]!

At the distance d equal to the functionality distance of 10 [m] the following design forces can be derived using the cost optimisation method,

$$F_{\mathrm{dy}} = 600 \text{ [kN]}$$

$$F_{\mathrm{dey}} = 428 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 1.40$$

$$F_{\mathrm{dey}} = 300 \text{ [kN]} \quad : \quad \varphi_{\mathrm{d}} = 2.00$$

For a distance d smaller than 7 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning.

At the distance $d = 5$ [m] the reduction in equivalent static design force about 10%.

5. Summary: The Conclusion

5.1 Highway lorry (parallel force)

The dynamic impact force parallel to the carriageway can be estimated by (force in [kN] and distance in [m]),

$$F_{\mathrm{dx}} = 2358 - 28 d_{\mathrm{const}}$$

without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) is in most cases **not safe to use**. For a distance d smaller than 40 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning. At the distance $d = d_{\text{fun}}$ the reduction in equivalent static design force is about 20%.

The dynamic impact force perpendicular to the carriageway can be estimated by, (force in [kN] and distance in [m]),

$$F_{\text{dy}} = 1508 - 18d_{\text{const}}$$

Without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) **is not safe to use**.

For a distance d smaller than 41 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning.

At the distance $d = d_{\text{fun}}$ the reduction in equivalent static design force is about 20%

5.2 Urban lorry (parallel force)

The dynamic impact force parallel to the carriageway can be estimated by (force in [kN] and distance in [m]),

$$F_{\text{dx}} = 1402 - 52d_{\text{const}}$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) is in most cases **safe to use**. The design force of 500 [kN] is ok for distances greater than 8 to 13 [m] (depending on the dynamic response of the pillar).

For a distance d smaller than 8 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning. At the distance $d = 5$ [m] the reduction in equivalent static design force is about 10%

The dynamic impact force perpendicular to the carriageway can be estimated by, (force in [kN] and distance in [m]),

$$F_{\mathrm{dy}} = 867 - 33d_{\mathrm{const}}$$

This indicates that without road restraint systems the equivalent static design force as mentioned in (EN 1991-2: Eurocode 1, 2003) **is not safe to use**.

For a distance d smaller than 7 [m] the economical distance d_{opt} is less than the strength distance d_{const}. This means that using design values smaller than the values derived from the strength constraint can be justified by economic reasoning.

At the distance $d = 5$ [m] the reduction in equivalent static design force is about 10%.

6. ACKNOWLEDGEMENTS

This research is supported by the IWT SBO project 60043,“Fuzzy Finite Element Method”.

7. References

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